**Statistics – Textbook Notes, Equations, Theorems**

**Chapter 1**

**Vocabulary**

1. Uncertainty
2. Variation
3. Samples
4. Populations
5. Statistical inference
6. Mean
7. Median
8. Standard deviations
9. Simple random sampling: any particular sample of a specified sample size has the same chance of being selected as any other sample size
10. Sample size: number of elements in the sample
11. Biased sample
12. Trimmed means: “trimming away” a certain percent of both the largest and the smallest set of values
13. Postulated model
14. Assumption
15. Normal distribution
16. Scatter plot
17. Stem and leaf plot: statistical data, generated in large masses, 🡪 presented in a combined tabular and graphic display
18. Frequency distribution: data, grouped into different classes and intervals, can be constructed by counting the leaves belonging to each stem and noting that each stem defines a class interval
19. Relative frequency histogram: use the midpoint of each interval and then its corresponding relative frequency
20. Distribution
21. Box and Whisker Plot or Box Plot: encloses interquartile range of the data in a box that has the median displayed within
    1. Upper and lower quartiles
    2. Whiskers are extreme observations

**Points:**

* The sample along with inferential statistics allows us to draw conclusions about the population, with inferential statistics making clear use of elements of probability
* Elements in probability allow us to draw conclusions about characteristics of hypothetical data taken from the population, based on known features of the population
* Two important parameters: population mean and population variance
  + **Sample variance** 🡪 role In method used to draw inferences about the population variance
  + Sample standard deviation and mean have important role in population mean
  + Standard deviation is used more in applications

**Equations:**

* Sample mean: numerical average of the sample at hand
* sample median: reflect the central tendency of the sample in such a way that its uninfluenced by extreme values or outliers

**Chapter 2 Probability**

**2.1 Sample Space**

* **chance outcomes**: presentation and interpretation of such outcomes that occur in a planned study or scientific investigation
* **categorical data**: numerical data, representing counts or measurements which can be classified according to some criteria
* three types of statistical studies: designed experiments, observational studies, and retrospective studies 🡪 end results = set of data that is subject to uncertainty
* *Definition 2.1:*
  + The set of all possible ooutcomes of a statistical experiement is called the **sample space** and is reperented by the symbol S
  + if the sample space has a finite number of members, we may LIST the members (separated by commas and enclosed by brackets)
* statement or rule method: the use of a vertical bar and when you have very large sets of data

**2.2 Events**

* **events**: more interested in specific occurances over the occurance of a specific element in the sample space
* *Definition 2.2*
  + An **event** is a subset of a sample space
* *Definition* *2.3*
  + The **complement** of a an event A with respect to S is the subset of all elements of S that are not in A. We denote the complement of A by the symbol A’
* *Definition 2.4*
  + The **intersection** of two events A and B, denoted by the symbol AB, is the event containing all elements that are common to A and B
* *Definition 2.5*
  + Two events A and B are **mutually exclusive**, or **disjoint**, if AB , that is, if A and B have no elements in common
* *Definition 2.6*
  + The **union** of the two events A and B, denoted by the symbol AB, is the even containing all the elements that belong to A or B or both

**2.3 Counting Sample Points**

* **Multiplication rule**: the fundamental principle of counting
  + Rule 2.1
    - If an operation can be preformed in ways, and if for each of these ways a second operation can be preformed in ways, then the two operations can be preformed together in ways
* **Generalized multiplication rule** 
  + Rule 2.2
    - If an operation can be preformed in ways, and if for each of these a second operation can be preformed in ways, and for each of the first two a third operation can be preformed in ways, and so forth, then the sequence of k operations can be performed in ways
* *Definition 2.7*:
  + A **permutation** is an arrangement of all or part of a set of objects
* *Definition 2.8*:
  + For any non negative integer , -- “, is defined as
    - Special case 🡪 !=1
* *Theorem 2.1:*
  + The number of permutation of n objects is
* *Theorem 2.2* 
  + The number of permutations of distinct objects taken at a time is
* *Theorem 2.3* 
  + The number of permutations of object arranged in a circle is
* *Theorem 2.4*
  + The number of distinct permutations of things of which are one of a kind, of a second kind, …, of a th kind is
* *theorem 2.5*
  + The number of ways of partitioning a set of objects into cells with elements in the first cell, elements in the second, and so forth, is
* **combinations**: where we are interested in the number of ways of selecting objects from without regard to order🡪 actually a partition with two cells, the one cell containing the objects and the other containing the objects that are left

**2.4 Probability of an Event**

* *Definition 2.9:* 
  + The probability of an event A is the sum of the weights of all sample points in A. Therefore,
  + furthermore, if is a sequence of mutually exclusive events then?
* If a sample space contains N elements, all of which are equally likely to occur, we assign a probability equal to 1/N to each of the N points. The probability of any event A containing of these N sample points is then the ratio of the number of elements in A to the number of elements in S
* Rule 2.3
  + If an experiment can result in any one of N differently equally likely outcomes, and if exactly of these outcomes correspond to event A, then the probability of event A is

**2.5 Additive Rules**

* its often easiest to calculate the probability of some event from known probabilities of other events 🡪 union of two other events or as the complement of some event
* **additive rule**: applies to the unions of events
* *Theorem 2.7*
  + If A and B are two events, then
  + Corollary 2.1:
    - If A and B are mutually exclusive
  + Corollary 2.2:
    - If are mutually exclusive, then
  + Corollary 2.3
    - If is a partition of sample space S, then
* *Theorem 2.8*
  + For three events A, B, and C
* *Theorem 2.9*
  + If A and A’ are complementary events, then

**2.6 Conditional Probability, Independence, and the Product Rule**

* when you’re interested in the probability structure under certain restrictions

**Conditional Probability**

* **Conditional probability**: probability of an event B occurring when its known that some event A has occurred
  + “the probability that B occurs given that A occurs”
* *Definition 2.10*
  + the conditional probability of B, given A, denoted by P(B|A), is defined bt
    - provided P(A) > 0

**Independent Events**

* **Independent events:** when the occurrence of one event has NO EFFECT on the occurrence of another
* *Definition 2.11*
  + Two events A and B are **independent** if and only if
    - or

assuming the existences of the conditional probabilities. Otherwise, A and B are **dependent**

**The Product Rule, or the Multiplicative Rule**

* enables us to calculate the probability that two events will **BOTH** occur
* *Theorem 2.10*
  + If in an experiment the events A and B can both occur, then
    - provided P(A)>0
* *Theorem 2.11*
  + Two events A and B are independent if and only if

therefore, to obtain the probability that two independent events will both occur, we simply find the product of their individual probabilities

* the multiplicative rule can be extended to more than two-event situation
* *Theorem 2.12*
  + If, in an experiment, the events can occure, then
  + if the events ,,…., are independent, then
* *Definition 2.12*
  + A collection of events are mutually independent if for any subset of A, for

**2.7 Bayes’ Rules**

**Total Probability**

* *Theorem 2.13*
  + If the event constitute a partition of the sample space S such that for i=1,2,…,k, then for any event A of S

**Bayes’ Rule**

* suppose that we select a product that was randomly selected and it is defective. What’s the probability that this product was made by machine
* *Theorem 2.14:*
  + If the events constitute a partition of the sample space S such that for then for any event A in S such that
    - for

**Chapter 3 – Random Variables and Probability Distributions**

**3.1 Concept of a Random Variable**

* *Definition 3.1*
  + A random variable is a function that associates a real number with each element in the sample space
* Capital X is used for the random variable; little letter x is one of the values
* *Definition 3.2* 
  + If a sample space contains a finite number of possibilities or an unending sequence with as many elements as there are whole numbers, it is called a **discrete sample space**
* *Definition 3.3*
  + If a sample space contains an infinite number of possibilities equal to the number of points on a line segment, its called a **continuous sample space**
* **Discrete random variable**: a random variable is called this if its set of possible outcomes I countable
* A random variable whose set of possible values is an entire interval of numbers is NOT discrete
* **Continuous random variable:** take on values on a continuous scale

**3.2 Discrete Probability Distributions**

* *Definition 3.4*
  + The set of ordered pairs is a **probability function, probability mass function, or probability distribution** of the discrete random variable X if, for each possible outcome x,
    - 1.
    - 2.
    - 3.
* *definition 3.5*
  + the **cumulative distribution function** of a discrete random variable X with probability distribution is

**3.3 Discrete Probability Distributions**

* when dealing with continuous variables, the f(x) is usually PDF
* *Definition 3.6* 
  + The function is a ***probability density function (pdf***) for the continuous random variable X, defined over the set of real numbers, if
    - 1.
    - 2.
    - 3.
* *definition 3.7*
  + the ***cumulative distribution function*** of a continuous random variable X with density function is
* as an immediate consequence of Def. 3.7

**3.4 Joint Probability Distributions**

* sometimes we want to record the simultaneous outcomes of several random variables
* **joint probability distribution:** if X and Y are two discrete random variables, the function would be
* *Definition 3.8*
  + The function is a ***joint probability distribution*** or probability mass function of the ***discrete*** random variables X and Y if
    - 1. for all (x, y)
    - 2.
    - 3.
  + for any region A in the xy plane,
* *definition 3.9* 
  + The function is a ***joint density function*** of the ***continuous*** random variables X and Y if
    - 1. for all
    - 2.
    - for any region A in the xy plane
* *definition 3.10*
  + The marginal distributions of X alone and of Y alone are
    - * for the discrete case, and
      * continuous
* *Definition 3.11*
  + Let X and Y be two random variables, discrete or continuous. The conditional distribution of the random variable Y given that X=x is
  + similarly, the conditional distribution of X given that Y=y is
* when it is known that the discrete variable Y=y, we evaluate
* when X and Y are continuous

**Statistical Independence**

* *Definition 3.12*
  + Let X and Y be two random variables, discrete or continuous, with joint probability distribution and marginal distributions and , respectively. The random variables X and Y are said to be statistically independent only if
* ***joint marginal distributions***
* *Definition 3.13*
  + Let be n random variables, discrete or continuous, with joint probability distribution and marginal distribution respectively. The random variables are said to be mutually statistically independent if and only if

**Chapter 4 Mathematical Expectation**

**4.1 Mean of a Random Variable**

* **Mean of random variable/mean of the probability distribution:** relative frequencies and we write it as or simply when its clear to which random variable we refer
* *Definition 4.1:* 
  + Let X be a random variable with probability distribution . The ***mean, or expected value,*** of X is
    - if X is DISCRETE
    - if X is CONTINUOUS
* *Theorem 4.1:* 
  + Let X be a random variable with probability distribution . The expected
  + value of the random variable is
    - if X is DISCRETE
    - if X is CONTINUOUS
* *Definition 4.2*
  + Let X and Y be random variables with joint probability distribution . The ***mean***, or expected value, of the random variable is
    - if X and Y are discrete
    - if X and Y are continuous

**4.2 Variance and Covariance of Random Variables**

* the mean, or expected value, of a random variable X is of special importance in statistics because it describes where the probability distribution is centered
* Variance of the random variable/ variance of the probability distribution X 🡪
* *Definition 4.3* 
  + Let X be a random variable with probability distribution and mean . The ***variance*** of X is
    - DISCRETE
    - CONTINUOUS
* ***deviation of an observation:*** 
  + since the deviations are squared and then averaged, will be much smaller of a set of x vales that are CLOSE to than it will be for a set of values that vary considerably from
* *Theorem 4.2*
  + The ***variance*** of a random variable X is
  + ***Discrete case***
  + for the continuous case the proof is step by step the same, with summations replaced by INTEGRATION
* *Theorem* *4.3*
  + Let X be a random variable with probability distribution . The ***variance*** of the random variable is
    - DISCRETE
    - CONTINUOUS
* *Definition 4.4* 
  + Let X and Y be random variables with joint probability distribution ***covariance*** of X and Y is
    - ***DISCRETE***
    - ***CONTINUOUS***
* *Theorem 4.4* 
  + The covariance of two random variables X and Y with means and , respectively, is given by
* although the covariance between two random variables does provide information regarding the nature of the relationship, the magnitude does NOT indicate anything regarding the strength of the relationship
* *Definition 4.5* 
  + Let X and Y be random variables with covariance and standard deviations and respectively. The ***correlation of X and Y*** is

**4.3 Means and Variances of Linear Combinations of Random Variables**

* *Theorem 4.5*
  + If an and b are constants, then
  + Corollary 4.1
    - Setting a=0, we see that E(b)=b
  + Corollary 4.2
    - Setting b=0, we see that E(aX)=(X)
* *Theorem 4.6*
  + The ***expected value*** of the sum or difference of two or more function of a random variable X is the sum or difference of the expected vales of the functions. That is
* *Theorem 4.7*
  + The ***expected value*** of the sum or difference of two or more function of the random variables X and Y is the sum or difference of the expected values of the functions. That is
  + Corollary 4.3
    - Setting we see that
  + Corollary 4.4
    - Setting we see that
* *Theorem 4.8* 
  + Let X and Y be two independent random variables, Then
  + corollary 4.5
    - Let X and Y be two independent random variables.
  + Corollary 4.6
    - Setting b=0, we see that
  + Corollary 4.7
    - Setting a = 1 and b = 0, we see that
  + Corollary 4.8
    - Setting b=0 and c=0, we see that
* Corollaries 4.6 and 4.7 state that the variance is UNCHANGED with ADD/SUB merely moves left/right. However, MULT/DIV then corollaries 4.6 and 4.8 state the variance is multiplied or divided by the square of the constant
  + Corollary 4.9
    - If X and Y are independent random variables, then
  + Corollary 4.10
    - If X and Y are independent random variables, then
      * -b is given here instead
  + Corollary 4.11
    - If are independent random variables, then

**Chapter 5 Some Discrete Probability Distributions**

**5.2 Binomial and Multinomial Distributions**

* **success or failure:** when an experiment often consists of repeated trials, each with two possible outcomes such as given
* **Bernoulli process:** when each trial is labeled as either a success or a failure

**The Bernoulli Process**

* Must possess the following *properties*
  + 1. The experiment consists of repeated trials
  + 2. each trial results in an outcome that may be classified as a success or a failure
  + 3. the probability of success, denoted by , remains constant from trial to trial
  + 4. The repeated trials are independent

**Binomial Distribution**

* **binomial random variable:** the number of successes in Bernoulli trials
* **binomial distribution:** the probability distribution of this discrete random variable
  + values denoted by 🡪 depend on number of trials, probability of a success on a given trial
* *Theorem 5.1* 
  + The mean and variance of the binomial distribution are
    - and

**Multinomial Experiments and the Multinomial Distribution**

* **multinomial experiment:** the binomial experiment becomes this if we let each trial have more than two possible outcomes.
  + Ex/ the drawing of a card from a deck with replacement is also a multinomial experiment if the 4 suits are the outcomes of interest
* In general, if a given trial can result in any one of possible outcomes with probabilities then the multinomial distribution will give the probability that occurs , occurs , etc.
* Since all the partitions are *mutually exclusive* and occur with *equal probability*, we obtain the multinomial distribution by *multiplying the probability for a specified order by the total number of partitions*
* ***Definition 🡪 equation*** 
  + If a given trial can result in the k outcomes with probabilities , then the probability distribution of the random variables in n independent trials, is
    - and

**5.3 Hypergeometric Distribution**

* done so without replacement 🡪 does NOT require replacement
* **Hypergeometric experiment:** when your interested in the probability of selecting x successes from the k items labeled successes and (n-x) failures from the (N-k) items labeled failures when a random sample of size n is selected from N items
  + two *properties*
    - 1. A random sample of size n is selected without replacement from N items
    - of the N items, k may be classified as successes and N-k are classified as failures
* **Hypergeometric random variable:** the number X of successes of a hypergeometric experiment
* **Hypergeometric distribution:** the probability distribution of the hypergeometric variable 🡪
* ***Definition*** 
  + The probability distribution of the **hypergeometric random variable** X, the number of successes in a random sample of size n selected from N items of which k are labeled success and N-k labeled failure, is
    - * The range of x can be determined by the three binomial coefficients in the definition, where x and (n-x) are no more than k and (N-k), respectively, and both of them cannot be less than 0
* *Theorem 5.2*
  + The mean and variance of the hypergeometric distribution are
    - and

**Relationship to the Binomial Distribution**

* the binomial distribution may be viewed as a large population version of the hypergeometric distribution
* comparing the mean and variance formulas of both
  + *variance differs (N-n)/(N-1) which is NEGLIGABLE when n is small relative to N*

**Multivariable Hypergeometric Distribution**

* If N items can be partitioned into the k cells with elements respectively, the the ***probability distribution of the random variables*** , representing the number of elements selected from in a random sample of size n, is
  + - with and

**5.4 Negative Binomial and Geometric Distributions**

* **negative binomial experiments:** we are now interested in the probability that the kth success occurs on the xth trial

**What Is the Negative Binomial Random Variable?**

* **Negative binomial random variable:** the number X of trials required to produce k successes in a negative binomial experiment
  + **Negative binomial distribution:** is its probability distribution
* ***What its probabilities depend on 🡪***
* The probability for the specified order ending in *SUCCESS* is
* total sample points in the experiment ending in a success after occurrence of (k-1) successes and (x-k) failures in any order, is equal to the number of partitions of (x-1) trials into two groups
  + (k-1) successes corresponding to one group
  + (x-k) failures corresponding to the other group
  + number specified by
    - mutually exclusive and occurring with equal probability
* ***Negative binomial distribution definition*** 
  + If repeated independent trials can result in a success with probability and a failure with probability , then the probability distribution of the random variable X, the number of the trial on which the kth success occurs, is
* ***SPECIAL CASE*** 
  + The **negative binomial distribution** where 🡪 probability distribution for the number of trials required for a SINGLE success
    - Ex/ tossing of a coin until a head occurs
  + the successive terms are a GEOMETRIC PROGRESSION 🡪 geometric distribution
* ***Geometric Distribution definition*** 
  + If repeated independent trials can result in a success with probability and a failure with probability then the probability distribution of the random variable X, the number of the trial on which the first success occurs, is
* *Theorem 5.3*
  + The **mean and variance** of a random variable following the geometric distribution are
    - and

**5.5 Poisson Distribution and the Poisson Process**

* **Poisson experiments:** an experiment that yields numerical values of a random variable, , the number of outcomes occurring during a given time interval or in a specified region
  + Time interval may be of any length

**Properties of the Poisson Process**

1. The number of outcomes occurring in one time interval or specified region of space is independent of the number that occur in any other disjoint time interval or region 🡪 Poisson distribution has NO MEMORY
2. The probability that a single outcome will occur during a very short time interval or in a small region is proportional to the length of the time interval or the size of the region and doesn’t depend on the number of outcomes occurring outside this time interval or region
3. The probability that more than one outcome will occur in such a short time interval or fall in such a small region is negligible

* **Poisson random variable:** the number of outcomes occurring during a Poisson experiment
  + **Poisson distribution:** its probability distribution
* **Definition and equation** 
  + The probability of the Poisson random variable , representing the number of outcomes occurring in a given time interval or specified region denoted by , is
  + Where is the average number of outcomes per unit time, distance, area r volume and
* *Theorem 5.4*
  + Both the **mean and the variance** of the Poisson distribution are

**Nature of the Poisson Probability Distribution**

* The form of the Poisson distribution becomes *more and more symmetric*, even bell shaped as the *mean grows larger*

**Approximation of Binomial Distribution by a Poisson Distribution**

* Poisson is related to Binomial…. limiting form of binomial
* If for Binomial, is quite large and is small, the conditions begin to simulate that of continuous space and time like Poisson
* If is larger and is close to 0, Poisson is used!
  + If is close to 1, we can still use it just would have to change the values for success and failure to make closer to 0
* *Theorem 5.5* 
  + Let be a **binomial random variable** with probability distribution . When and remains constant

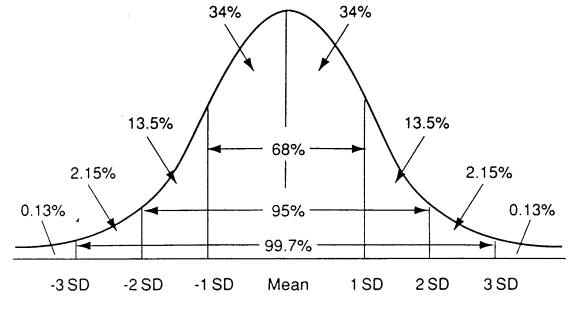
**Chapter 6 Some Continuous Probability Distributions**

**6.1 Continuous Uniform Distributions**

* **continuous uniform distribution:** characterized by a density function that is “flat” and thus the probability is uniform in a closed interval, say
* **Uniform Distribution definition** 
  + The density function of the continuous uniform random variable on the interval is
  + the density function forms a rectangle with base and **constant height** . 🡪 **rectangular distribution**
    - interval may NOT always be closed
* *Theorem 6.1* 
  + The **mean and variance** of the uniform distribution are

**6.2 Normal Distribution**

* **normal distribution/Gaussian distribution:** most important
  + **normal curve:** is the bell shaped curve which approximately describes many phenomena that occur in nature, industry and nature



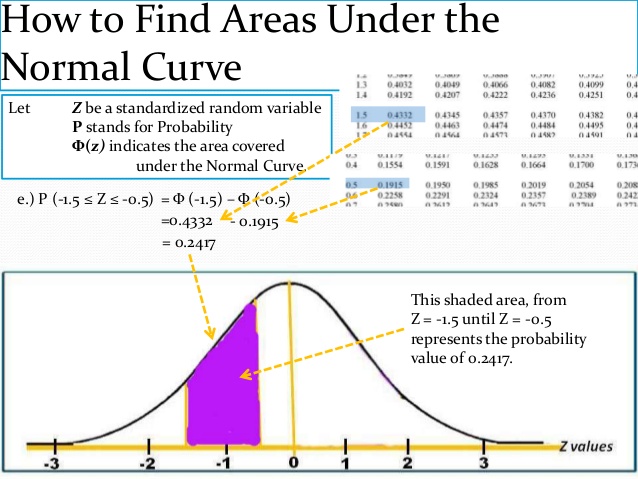
* **Normal Distribution definition**
  + **The density of the normal variable** , with mean and variance is
* if two normal distributions have lets say the SAME STANDARD DEVIATION but DIFFERENT MEANS 🡪 they will be the same form/height/shape but centered at different locations
* lets say two normal curves have the SAME MEAN but DIFFERENT STANDARD DEVIATION 🡪 the curve with the larger standard deviation will be MORE SPREAD OUT and lower
* *Properties of the Normal Curve:*

1. The mode, which is the pint on the horizontal axis where the curve is a maximum, occurs at
2. The curve is symmetric about a vertical axis through the mean
3. The curve has its points of inflection at ;
   1. Concave down 🡪
   2. Concave up 🡪
4. The normal curve approaches the horizontal axis asymmetrically as we process in either direction away from the mean
5. The total area under the curve and above the horizontal axis is equal to 1

* *Theorem 6.2:*
  + The **mean and variance** of are and . Hence the standard deviation is
* The normal distribution finds enormous applications as a “limiting distribution”

**6.3 Areas under the Normal Curve**

* The curve of any continuous distribution or density function is constructed so that the *area under the curve bounded by two ordinates and equals the probability that the random variable*  assumes a value between and



* Difficulty encountered in solving integrals of normal density functions necessitates the tabulation of normal curve areas for quick reference. \
  + We are able to transform all the observation of any normal random variable into a new set of observation of a normal random variable with mean 0 and 1
  + If
* *Definition 6.1*
  + The distribution of a normal random variable with mean 0 and variance 1 is called a **standard normal distribution**

**Using the Normal Curve in Reverse**

* Sometimes we are required to find the value of corresponding to a specified probability that falls between values listed in Table A.3
  + 🡪 choose value closes to the specified probability given
    - to give 𝑥

**6.5 Normal Approximation to the Binomial**

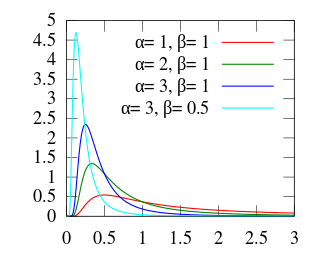
* the normal distribution is often a good approximation to a discrete distribution when the latter takes on a symmetric bell shape
* below is a theorem which allows to use areas under the normal curve to approximate binomial properties when is sufficiently large
* *Theorem 6.3* 
  + If is a binomial random variable with mean and variance , then the **limiting form of the distribution** of
  + as , is the the standard normal distribution

**6.6 Gamma and Exponential Distributions**

* the exponential and gamma distributions play an important role I both queuing theory and reliability problems
* *Definition 6.2 (Gamma)*
  + The **gamma function** is defined by
* *Properties of gamma function*

1. , for a positive integer
2. for positive integer

* **Gamma Distribution definition** 
  + The continuous random variable has a **gamma distribution**, with parameters and , if its density function is given by
    - * where



* **Exponential Distribution (definition)**
  + The continuous variable has an **exponential distribution**, with parameter , if its density function is given by
    - * where
* the following theorem and corollary give the mean and variance of the gamma and exponential distributions
* *theorem 6.4*
  + the **mean and variance** of the gamma distribution are
    - and
  + Corollary 6.1
    - The **mean and variance** of the exponential distribution are
      * and

**The Memorials Property and Its Effect on the Exponential Distribution**

* the types of application of the exponential distribution in reliability and component or machine problems are influences by the **memoryless** (or lack of memory) **property** of the exponential function

**6.7 Chi-Squared Distribution**

* **chi-squared distribution:** another very important special case of the gamma distribution is obtained by letting , where is a POSITIVE integer
  + degree of freedom (
* **Chi-Squared Distribution (definition)** 
  + The continuous random variable has **chi-squared distribution**, with degress of freedom, if its **density function** is given by
* *Theorem 6.5* 
  + The **mean and variance** of the chi-squared distribution are
    - and

**6.8 Beta Distribution**

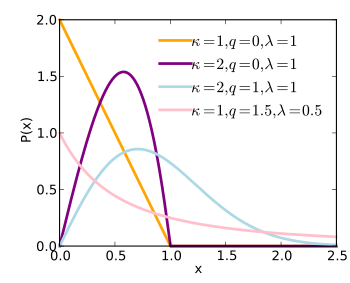
* an extension to the uniform distribution is a beta distribution
* *Definition 6.3*
  + A **beta function** is defined by
    - * where is the gamma function
* **Beta Distribution (definition eqn)**
  + The continuous random variable has a **beta distribution** with parameters and of its density function is given by
* *Theorem 6.6* 
  + The **mean and variance** of a beta distribution with parameters and are
    - and

**6.9 Lognormal Distribution**

* this distribution applies in cases where a natural log transformation results in a normal distribution
* **Lognormal Distribution (definition)** 
  + The continuous random variable has a **lognormal distribution** if the random variable has a normal distribution with mean and standard deviation . The resulting **density function** of X is
* *Theorem 6.7*
  + The **mean and variation** of the lognormal distribution are

**6.10 Weibull Distribution**

* **Weibull Distribution (definition)**
  + The continuous random variable has a **Weibull distribution**, with parameters and , if its **density function** is given by
    - * where and
* *Theorem 6.8* 
  + The **mean and variance** of Weibull distribution are



* **cdf for Weibull Distribution** 
  + The **Cumulative distribution function** for the Weibull distribution is given by
* **Failure Rate for Weibull Distribution** 
  + The **failure rate** at time for the Weibull distribution is given by

**Interpretation of the Failure Rate**

* The quantity is aptly named as a failure rate since it does quantify the rate of change over time of the conditional probability that the component lasts an additional “given that it has lasted to time t”

1. If , that failure rate = , a constant. This, as indicated earlier, is the special case of the exponential distribution in which lack of memory prevails
2. If , is an increasing function of time , which indicated that the component wears over time
3. If , is a decreasing function of time and hene the component strengthens or hardens over time